ASIAN JOURNAL OF CIVIL ENGINEERING (BUILDING AND HOUSING) VOL. 3, NO. 1,(2002)
PAGES 1-12

SPLINE FINITE MEMBER ELEMENT METHOD FOR LATERAL BUCKLING OF THIN-WALLED MEMBERS WITH OPENINGS

Quanfeng Wang
Department of Civil Engineering, Huaqiao University, 362011 Quanzhou, Fujian, China

ABSTRACT

The present study is focused on establishing a numerical procedure, based on energy principle, to estimate the lateral buckling capacity of thin-walled members with openings. The numerical method (spline finite member element method) proposed in this paper combines the advantages of B_3-spline, the finite element method and Vlasov's thin-walled beam theory. In this method the effects of torsion, warping and the shearing strains in middle surface of the walls are taken into account. Compared to the experimental results, the numerical results presented in this paper demonstrate the efficiency of the proposed method. Since there is no need to introduce the concepts of shear centre and sectorial co-ordinate, the method can be easily understood and applied. It is concluded that the energy equation used for lateral buckling analysis is applicable to thin-walled members with an arbitrarycross section.

Keywords: lateral buckling, spline finite members, thin walled members.

1. INTRODUCTION

As far as the lateral buckling of thin-walled members is concerned, the buckling usually occurs by twisting or by a combination of bending and twisting. Therefore, the buckling of thin-walled member differs from Euler's classical theory of lateral buckling and is much more complex. This lateral buckling is of importance in the design of structures because thin-walled members have been widely used in civil engineering. Questions which often arise in design, are those related to problems arising when large openings are cut in the members because these are frequently required in buildings for the passage of service ducting and piping, and in bridge structures for inspection purposes. It is therefore, of practical significance to develop a consistent method for lateral buckling analysis of thin-walled members with openings.

The problem of the buckling of thin-walled members has attracted many investigators during the past few decades. The influences of load form, general member dimensions, and support conditions on the buckling have been extensively studied [1]. But comparatively little work appears to have been carried out on the influence of openings on the buckling behaviour. The studies reported by Thevendran and Shanmugam [2], and Wang [3] indicate that the numerical methods, using the principle of minimum total potential energy and

weighted residuals method, can be respectively used to predict the lateral buckling behaviour of cantilever and supported beams. The results are presented only in view of the data of double symmetrical cross sections. Tests were carried out to investigate the effect of opening on lateral buckling of beams using a modified "Southwell Plot" [4]. Although the results refer only to slender cantilever beams with symmetrical cross section, an effective stiffness concept has been shown to yield a reasonable approximate prediction of the buckling load. In fact, a few attempts have been made to treat the generalized stability on the basis of computer analysis. To the best of author's knowledge, no simple theory comparable to the Timoshenko or Vlasov theory has been developed to deal with lateral buckling of thin-walled members with openings. The finite element method was applied to this problem, but finite element analysis of such structures is quite expensive, even when planar formulation is adopted, and unless very fine meshes are used, they are not likely to be more accurate than some of the continuum methods [5]. It is, therefore, of practical significance to develop a consistent method for thin-walled members with any cross section. The present study is focused on establishing a numerical procedure, based on energy principle, to estimate the lateral buckling capacity of thin-walled members with openings. The numerical method (spline finite member element method) proposed in this paper combines the advantages of B_3-spline, the finite element method and Vlasov's thin-walled beam theory [6]. The displacements at the two ends of the member element are adopted as basic variables, and a transformed B_3-spline function is used to simulate the longitudinal warping displacement field along the cross section of thin-walled member. In the numerical method, the effects of torsion, warping and the shearing strains in middle surface of the walls are taken into account.

Compared to the experimental results, the numerical results proposed in the paper demonstrate the efficiency of the proposed method. However, by using the proposed procedure a great saving in computer time and data preparation effort is achieved, when compared to the finite element procedure.

2. ENERGY EQUATION OF THIN-WALLED MEMBERS

The energy criterion of elastic stability is characterised by the buckling equation in which the total potential energy ceases to be a minimum. Naturally, the criterion is restricted to conservative system and within a linearized analysis it leads to an eigenvalue determinant from which the critical buckling load is retrieved. Using numerical method to slove this eigenvalue problem,the thin-walled is replaced by a discrete member element model. The member element of a prismatic thin-walled member with arbitrary cross section subjected to lateral loading is shown in Fig.1, in which the z axis is the longitudinal axis, the x and y axes are the principal axes passing through the centroid c of the cross section; s is the curvilinear coordinate along the central line of the cross section; and H is the longitudinal length of the member element.

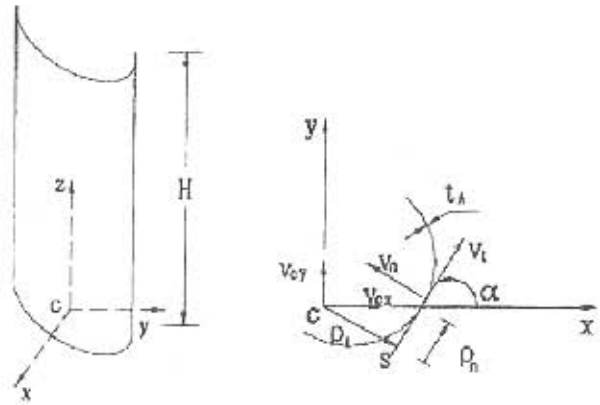

Figure 1 Thin-walled member with arbitrary cross section and its parameters

The member element stiffness matrix can be derived by applying the principle of stationary potential energy. For a thin-walled member, the total potential energy of a general member element of length H is summation of strain energy U and loading potential energy V given by the following integrals [7].

$$\Pi = U + V = \frac{1}{2}\int_0^H \left\{ \int_{\Sigma s} \left[E(\frac{\partial w}{\partial z})^2 t_o + G(\frac{\partial w}{\partial s} + \frac{\partial v_t}{\partial z})^2 t \right] ds + GJ_d (\frac{d\theta}{dz})^2 \right\} dz$$
$$-\frac{1}{2}\int_0^H \int_{\Sigma s} \sigma \left[(\frac{\partial w}{\partial z})^2 t_o + (\frac{\partial v_x}{\partial z})^2 t + (\frac{\partial v_y}{\partial z})^2 t \right] ds\ dz \tag{1}$$
$$-\frac{1}{2}\int_0^H \int_{\Sigma s} \tau \frac{\partial v_n}{\partial s} \frac{\partial v_n}{\partial z}\ t\ ds\ dz$$

in which E is the Young modulus; G is the shear modulus; J_d is the St. Venant's torsional constant; $w(s,z)$ is the longitudinal displacement along z direction; v_t and v_n are the displacements along the tangent and normal direction of the central line at point s; Σs is the whole length of cross-section; θ is the twisting angle of the cross section; σ and τ are the normal and shear stresses at any point prior to buckling due to applied load system; v_x and v_y are the displacement of any point of cross section in the x and y directions, respectively. t_o is the actual thickness of wall at point s; and t is the equivalent thickness of wall when there is a row of holes in the wall. In the paper, only one of the two Valso'v assumptions [6], i.e. "rigid cross section" is retained, the tangent displacement can be expressed by the centroid displacement.

$$v_t = [\eta_t]_{1x3}\{v_c\}_{3x1}; \tag{2a}$$
$$v_n = [\eta_n]_{1x3}\{v_c\}_{3x1}; \tag{2b}$$

in which $[\eta_t] = [\cos\alpha\ \ \sin\alpha\ \ \rho_t]$; $[\eta_n] = [-\sin\alpha\ \ \rho_n]$; $\{v_c\} = [v_{cx}\ \ v_{cy}\ \ 0]^T$; α is the angle between

x axis and tangent at point s; ρ_t is the distance from c to the tangent of point s; ρ_n is the distance from point s to ρ_t; v_{cx} , v_{cy} are x and y direction components of the centroid displacement v_c , respectively.

3. LONGITUDINAL WARPING DISPLACEMENT FIELD

The spline function is chosen in this paper to represent the longitudinal warping displacement field on the segments of cross section. To extend the applied range of the proposed method, this spline function is modified here. The warping displacement $w(s,z)$ is taken as the summation of $(w+3)$ local B_3-splines by

$$w(s,z) = \sum_{i=l}^{m+3} \alpha_i(z)\psi_i(s) \tag{3}$$

in which s is the curvilinear coordinate along the central line of the cross section; α_i is the knot generalized displacement parameters; m is number of equal subintervals of a segment of the cross section, and ψ_i is an ordinary local B_3-spline. Let w_j be the real knot displacement in the segment, i.e.

$$\begin{cases} w_1(z) = \dfrac{\partial w(s_2,z)}{\partial s}, \\ w_2(z) = w(s_2,z), \\ w_i(z) = w(s_i,z), \\ w_{m+2}(z) = w(s_{m+2},z), \\ w_{m+3}(z) = \dfrac{\partial w(s_{m+2},z)}{\partial s} \end{cases} \tag{4}$$

in which s_2 and s_{m+2} are the coordinates of the beginning and the end knots in segment, repectively. Because of the localization of the spline function, Eq.3 can be expressed by m interval function.

$$w_j(s,z) = \sum_{i=1}^{m+3} \alpha_i(z)\psi_i(s)(s_{j-1} \le s \le s_{j+2}; j = 1,2.3,...m) \tag{5}$$

Solving the above simultaneous equations 4 and 5 by the package "Maple V" [8], the knot generalized displacement parameters α_i (i = 1,2, … , m+3) can be expressed by real knot displacements w_i as follows:

$$\alpha_i = \sum_{k=1}^{m+3} b_{ik} w_k (i = 1,2,...,m+3)$$

in which b_{ik} are the coefficients computed by means of the package "Maple V".

Substituting the above equation into Eq.5 gives:

$$w_j(s,z) = \sum_{k=1}^{m+3} \overline{\psi}_{jk}(s)_{w_k}(z) \ \ (s_{j-1} \le s_{j+2}; j = 1,2,3...m) \tag{6}$$

in which

$$\overline{\psi}_{jk}(s) = \sum_{i=j-2}^{j+1} b_{ik}\psi_i(s),$$

By means of the transformed spline functions the longitudinal warping displacements of the whole cross section can be expressed as

$$w_j(s,z) = \sum_{i=1}^{n} \overline{\psi}_{ji}(s) w_i(z) = [\overline{\psi}]_{1xn}\{w\}_{nx1} \tag{7}$$

in which n denotes the total number of knots dividing the whole cross section into subintervals, and

$$[\overline{\psi}]_{1xn} = [\overline{\psi}_{j1}(s), \overline{\psi}_{j2}(s)...\overline{\psi}_{jn}(s)]; \{w\}_{nx1} = [w_1(z), w_2(z)...w_n(z)]^T.$$

4. BUCKLING ANALYSIS OF THIN-WALLED MEMBERS

Providing that the x and y axes are the principal axes passing through the centroid c of a cross section, v_x and v_y are related to v_{cx}, v_{cy} and θ by

$$\begin{cases} v_x = v_{cx} - y\theta, \\ v_y = v_{cy} + x\theta; \end{cases} \tag{8}$$

Substituting Eqs.2, 7 and 8 into Eq.1 gives

$$\begin{aligned} \frac{1}{2}\int_0^H \Big\{ & (E\{w'\}^T[A]\{w'\} + G\{w\}^T[B]\{w\} + 2G\{w\}^T[C]\{v_{c'} \\ & + G\{v_{c'}\}^T[D_t]\{v_{c'}\} + G\{\theta'\}^T J_d\{\theta'\}) \\ & - (\{w'\}^T[T_{c1}]\{w'\} + \{v_{c'}\}^T \int_{\Sigma s} \sigma[T_\sigma] t\, ds\{v_{c'}\} \\ & + \{v_{c'}\}^T \int_{\Sigma s} \tau[T_\tau] t\, ds\{v_c\} + \{v_c\}^T \int_{\Sigma s} \tau[T_\tau]^T t\, ds\{v_{c'}\}) \Big\} dz = 0 \end{aligned} \tag{9}$$

in which

$$[A]_{nxn} = \int_{\Sigma s} [\overline{\psi}]^T [\overline{\psi}] t\, d\, s; [B]_{nxn} = \int_{\Sigma s} [\overline{\psi}']^T [\overline{\psi}'] t\, d\, s;$$

$$[C]_{nx3} = \int_{\Sigma s} [\psi']^T [\eta_t] t\, d\, s; [D_t]_{3x3} = \int_{\Sigma s} [\eta_t]^T [\eta_t] t\, d\, s;$$

$$[T_{c1}]_{nxn} = \int_{\Sigma s} \sigma [\overline{\psi}]^T [\overline{\psi}] t\, d\, s;$$

$$[T_\sigma]_{3x3} = \begin{bmatrix} 1 & 0 & -y \\ 0 & 1 & x \\ -y & x & x^2+y^2 \end{bmatrix}; [T_\tau]_{3x3} = \begin{bmatrix} 0 & 0 & -\sin\alpha \\ 0 & 0 & \cos\alpha \\ 0 & 0 & x\cos\alpha + y\sin\alpha \end{bmatrix};$$

Performing the integral over the cross section and noting that x and y are the principal axes passing through the centroid equation yields:

$$\int_{\Sigma s} \sigma [T_\sigma] t ds = \begin{bmatrix} 0 & 0 & -M_x \\ 0 & 0 & M_y \\ -M_x & M_y & \Delta_\sigma \end{bmatrix} = [T_M]_{3x3}$$

$$\int_{\Sigma s} \tau [T_{tauHs}] t ds = \begin{bmatrix} 0 & 0 & -Q_y \\ 0 & 0 & Q_x \\ 0 & 0 & \Delta_Q \end{bmatrix} = [T_Q]_{3x3}$$

in which,

$$\Delta_\sigma = \int_{\Sigma s} \sigma (x^2+y^2) t ds; \Delta_\tau = \int_{\Sigma s} \tau (x\cos\alpha + y\sin\alpha) t ds;$$

and M_x and M_y are the bending moments of lateral load about the principal axes x and y respectively; Q_x and Q_y are the corresponding shear force. Referring to Eq.9, the non-linear terms are:

$$V = -\frac{1}{2}\int_0^{l'} \Big(\{v_{c'}\}^T [T_M] \{v_{c'}\} + \{w'\}^T [T_{c1}] \{w'\} + \{v_{c'}\}^T [T_Q] \{v_c\} + \{v_c\}^T [T_Q]^T \{v_{c'}\} \Big) dz; \tag{10}$$

By variation with respect to $\{w\}$ and $\{v_c\}$, and using some standard techniques such as integration by parts in the derivation in Eq.9, the following differential equations can be obtained:

$$E(A)\{w''\} - G[\overline{B}]\{w\} = -G[C]\big(G[D] - [T_M]\big)^{-1} [T_Q]\{v_c\} + G^2 [C][T_M]^{-1} [C]^T [w] + [T_{c1}]\{w''\} + G[C]\{w'\}\{v_c\}, \tag{11}$$

and boundary conditions

$$E[A]\{w'\}-[T_{c1}]\{w'\}=\{0\}; \tag{12}$$

$$G[C]^T\{w\}+G[D]\{v_{c'}\}-[T_M]\{v_{c'}\}-[T_Q]\{v_c\}-([T_Q])^T\{v_{c'}\}=\{0\} \tag{13}$$

in which

$$[\bar{B}]=[B]-[C][D]^{-1}[C]^T;$$

$$[D]_{3x3}=[D_t]+J_d\begin{bmatrix}0&0&0\\0&0&0\\0&0&1\end{bmatrix};$$

5. SOLUTION OF BUCKLING EQUATION

From the "rigid cross section" assumption, the displacements at one end of the member element are only the lateral displacements and twisting angle of the cross section and n longitudinal displacements of knots. In matrix form, the end displacements of the member element expressed by a vector {WE} as:

$$\{W_E\}_{(2n+6)x1}=[v_{1x},v_{1y},\theta_1,w_{11},w_{12}...w_{1n},v_{2x},v_{2y},\theta_2,w_{21},w_{22}...w_{2n}]^T \tag{14}$$

in which the first subscript 1 or 2 denotes first or second end cross section; v_{1x} denotes the lateral displacement of the cross section in the x direction at first end, w_{11} denotes the longitudinal displacement of first knot at first end, etc. Note that n is an arbitrary chosen number of knots by which the cross section is divided into subintervals to fulfil the accuracy requirements of analysis for different shapes of cross sections. Similar to the finite element analysis, a sline finite member element can be established for the thin-walled structure analysis. Define a transformation as

$$\{w\}_{nx1}]=[a]_{nxn}\{U\}_{nx1} \tag{15}$$

in which

$$[a]=[\{a_1\},\{a_2\},...,\{a_n\}],\{U\}=[U_1,U_2,...,U_n]^T,$$

and $\{a_i\}$ is an eigenvector. From past studies [7], the following displacement vector can be expressed in terms of end displacements of member element:

$$\begin{Bmatrix}v_c\\w\end{Bmatrix}_{(n+3)x1}=\begin{pmatrix}[T_v(z)]_{3x(2n+6)}\\[a]_{nxn}[T_w(z)]_{nx(2n+6)}\end{pmatrix}[T_E]^{-1}_{(2n+6)x(2n+6)}\{W_E\}_{(2n+6)x1}; \tag{16}$$

in which,

$$[T_v(z)]_{3x\{2n+6\}} = \Big[-[D]^{-1}_{3x3}[C]^T_{3xn}[a]_{nxn}\int_0^z[R_s]_{nxn}dz[a]^T_{nxn}[C]_{nx3}$$

$$+z\frac{G}{E}[I]_{3x3},[I]_{3x3},[D]^{-1}_{3x3}[C]^T_{3xn}[a]_{nxn}\int_0^z[R]_{nx2n}dz\Big];$$

$$[T_w(z)]_{nx\{2n+6\}} = \Big[[R_s]_{nxn}[a]^T_{nxn}[C]_{nx3},[0]_{nx3},[R]_{nx2n}\Big];$$

$$[T_E]_{\{2n+6\}x\{2n+6\}} = \begin{bmatrix} [T_v(0)]_{3x\{2n+6\}} \\ [a]_{nxn}[T_w(0)]_{nx\{2n+6\}} \\ [T_v(H_i)]_{3x\{2n+6\}} \\ [a]_{nxn}[T_w(H_i)]_{nx\{2n+6\}} \end{bmatrix},$$

$$[R_s]_{nxn} = \begin{bmatrix} \frac{z^2}{2} & & & & & & \\ & \frac{z^2}{2} & & & 0 & & \\ & & \frac{z^2}{2} & & & & \\ & & & -\frac{1}{\Lambda_4}\frac{E}{G} & & & \\ & & & & \cdot & & \\ & & & & & \cdot & \\ & 0 & & & & \cdot & \\ & & & & & & -\frac{1}{\Lambda}\frac{E}{G} \end{bmatrix}$$

$$[R]_{nx2n} = \begin{bmatrix} z, 1 & & & & & & \\ & z, 1 & & & 0 & & \\ & & z, 1 & & & & \\ & & & e^{-\lambda_4\xi}, & e^{-\lambda_4\eta} & & \\ & & & & \cdot\ \cdot & & \\ & & & & & \cdot\ \cdot & \\ & 0 & & & & \cdot\ \cdot & \\ & & & & & & e^{-\lambda_4\xi}, \quad e^{-\lambda_4\eta} \end{bmatrix}$$

$$\eta = \frac{z}{H_i}, \xi = 1-\eta, \lambda_i = H_i\sqrt{\frac{G}{E}\Lambda_i}$$

and $[I]$ is a unit diagonal matrix; Λ_1, Λ_2, ..., Λ_n are the eigenvalues of the following

eigenvalue equation:

$$\Lambda[A]_{nxn}\{a\}_{nx1} - [\overline{B}]_{nxn}\{a\}_{nx1} = \{0\}_{nx1} \tag{17}$$

Substituting Eq.16 into Eq.11 yields the following standard formula of finite element stiffness equation for buckling analysis:

$$([K]+[K_G])_{(2n+6)x(2n+6)}\{W_E\}_{(2n+6)x1} = \{0\}_{(2n+6)x1} \tag{18}$$

in which $[K]$ is the linear strain stiffness matrix which comes from Eq.11 when $\sigma = 0$. Following the standard procedures of the finite element method, the stiffness matrix $[K]$ of the spline member element can be obtained; $[K_G]$ is referred to as the non-linear geometric stiffness matrix and can be derived from Eq.10

$$V = -\frac{1}{2}\{W_E\}^T_{1x(2n+6)}[K_G]_{(2n+6)x(2n+6)}\{W_E\}_{(2n+6)x1} \tag{19}$$

and written by

$$[K_G] = \left([T_E]^{-1}\right)\int_0^H \left([T_{v'}(z)]^T[T_M][T_{v'}(z)] + \{T_{v'}(z)\}^T[T_Q]\{T_v(z)\} + \{T_v(z)\}^T[T_Q]^T\{T_{v'}(z)\}\right)dz[T_E]^{-1};$$

6. EFFECT OF OPENINGS ON LATERAL BUCKLING

Since the presence of openings may be regarded as roughly similar to a reduction in the effective thickness, a possible approximate evaluation of the buckling load for a member with openings may be derived by replacing the thickness by an effective thickness which takes account of the loss of material. This may be achieved by considering the horizontal portion of the member containing the openings to act as a segment with a reduced thickness, by "diffusing" the area of web sections between openings throughout the whole length of the member. In that case, the buckling load becomes [4]

$$P_{cro} = a_j P_{cr} \tag{20}$$

in which j is either 1 or 2; the reduced coefficient of rigidity due to rectangular openings becomes.

$$\alpha_1 = (1.\frac{b_o}{d}) + (1.\frac{a_o}{L})^3\frac{b_o}{d}, \tag{21}$$

n_o is the number of rectangular openings; a_o is the length of opening; b_o is the depth of opening, and d is the overall web depth, L is the length of the member. For members with circular openings, a corresponding approximate formula is very complicated in view of the irregularity of the web areas between openings. Since the effective width concept is relatively

crude, the derivation of an accurate formula is not warranted, and a simple solution may be achieved by replacing the circular opening by an equivalent polygonal opening. The simplest approach is to replace the real circular opening by an octagonal opening, the polygon being circumscribed in a circle of diameter D_o. The reduced coefficient then becomes

$$\alpha_2 = (1.\frac{D_o}{d}) + 0.172(\frac{n_o D_o^2}{dL} + \frac{D_o}{d})(1. - \frac{n_o D_o}{L}) \tag{22}$$

7. COMPARISON WITH EXPERIMENTAL RESULTS

The following example has been investigated by Thevendran and Shanmugam [2]. And, it can be observed later in the section that the proposed numerical analysis for the lateral buckling load in this paper is very efficient.

Example:
The specimens of simply supported I-beam are of length L=940mm, web thickness t_w=6mm, overall web depth d=75mm, flange thickness t_f=10mm, and flange width b=23.5mm as shown in Figure.2.

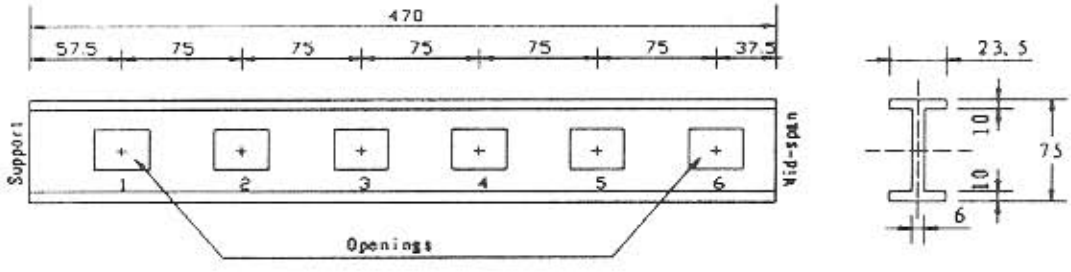

Figure 2 Locations of openings

The locations for the openings over the left half of the span are indicated in Fig.2. The six locations are numbered from the support to mid-span, and are symmetric about mid-span. The openings are either rectangular or circular, with their number and sizes varied. Specimens with the following sets of openings are studied: 1. No opening; 2. One rectangular opening; 3. Three rectangular openings; 4. Six rectangular openings; 5. Three circular openings. For the sets 2,3, and 4, two different sizes are considered, 62.6mm X 50mm and 62.5mm X 25mm; For 5, two different diameters, 38mm and 2mm, are considered. The test is carried out using specimens from plexiglass sheets having average value of Young's modulus E=2.860 N/mm^2 and Poison's ratio=0.36. The numerically computed lateral buckling capacity of the member is compared with those obtained from experimental results [2] in table 1. From this table, it can be seen that the lateral buckling loads of P_{cra} obtained by using the proposed method in this paper are close to the experimental results P_{cre}. In most cases the numerical results differ within 9% from the corresponding experimental results.

Table 1 Comparison of results of simply supported rectangular members (N)

Specimen	Size of Opening (mm)	Location of Opening	P_{cre} by Experiment	P_{cra} by Proposed M.	P_{cre}/P_{cra}
SR0RA	---	---		89.8	0.86
SR1RF	62.5 X 50.	6	59.3	68.9	1.02
SR1RI	62.5 X 25.	6	80.9	79.4	1.23
SR3RA	62.5 X 50.	1,3,6	52.7	42.9	1.11
SR3RB	62.5 X 25.	1,3,	74.0	66.4	1.01
SR6RA	12.5 X 25.	1-6	78.5	77.6	0.99
SR6RB	25.0 X 25.	1-6	68.7	69.3	1.02
SR6RC	37.5 X 25.	1-6	65.7	64.1	0.95
SR6RD	50.0 X 25.	1-6	58.0	61.3	0.91
SR6RE	62.5 X 25.	1-6	54.9	60.1	0.93
SR6RF	62.5 X 12.5	1-6	69.9	75.0	0.97
SR6RG	62.5 X 31.0	1-6	51.5	52.9	1.06
SR6RH	62.5 X 37.5	1-6	48.1	45.3	1.03
SR6RI	62.5 X 50.0	1-6	31.4	30.4	0.99
SR3CA	D=38.0	1,3,6	80.5	80.6	1.01
SR3CB	D-25.0	1,3,6	86.8	85.8	

8. CONCLUSIONS

A spline finite member element method, based on the displacement variation principle, is developed for lateral buckling anaylsis of thin-walled members with openings. According to the above discussions, the following conclusions can be drawn: (a). The energy equation for lateral buckling analysis is applicable for thin-walled members with any cross section; (b). The proposed method has an advantage over the methods based on the classical theory because the effect of the shearing strains of the middle surface of walls, which reflects the shear lag phenomenon, on the buckling is considered. (c). Since there is no need to introduce the concepts of shear centre and sectorial co-ordinate, the method is easily understood and applied. (d). Compared with experimental results, numerical examples presented this paper demonstrate the efficiency of the proposed method.

REFERENCES

1. Pin, Young-Lin, and Trahair, N.S. "Prebuckling deflections and lateral buckling I: theory". *Journal of Structural Engineering, ASCE*, No. 11, **118**(1992) 1462-1479.
2. Thevendran, V., and Shanmugam, N. E. "Lateral buckling of doubly symmetric beams containing openings". *Journal of Engineering Mechanics, ASCE*, (1991) 1427-1441.
3. Wang, Quanfeng. "Lateral buckling of thin-walled members with openings considering shear lag". *An Int. J., Structural Engineering and Mechanics*, **5**(4) (1997) 369-384.
4. Coull, A., and Alvarez, M.C. "Effect of openings on lateral buckling of beams". *Journal of the Structural Div., ASCE*, **106**(12) (1980) 2553-2560.
5. Rutenberg, A. V., and Eisenberger, M. "Torsion of tube structures: planar formulation". *Computers and Structures*, **17**(1983) 257-260.
6. Vlasov, V.Z. Thin-Walled Elastic Beams. 2nd Ed. Program for Scientific Translation, 1961.
7. Wang, Quanfeng, and Li, W.Y. "Lateral buckling of thin-walled members with shear lag using spline finite member element method". *Computers and Structures*, **75**(1) (2000) 81-91.
8. Bruce, W.C. et al, Maple Language Reference Manual, Waterloo Maple Publisher, 1991.